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MONEY, THE ECONOMIC VEIL

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1.

Wicksell's "natural rate of interest" has been explained and defined in a variety of ways[1], but some of the explanations advanced are unsupportable. One of the theories which I am unable to accept is the explanation that the natural rate of interest is the rate of interest which is formed wherever capital goods are lent or borrowed in kind, that is, in the exchange of goods.

An exponent of this theory regards the natural rate of interest thus defined as being identical with the rate involved where savings are in demand for investment; which would make savings equal investment. Such a view clearly implies the concept that money is merely a sort of veil covering but not affecting value. This approach, to anticipate my conclusion, is one which we cannot in any sense support. In any case it can hardly be denied that the rate of interest is influenced by the interposition of money into the social economy, consequently it must differ in nature from a rate derived from the mere exchange of goods in kind. This consideration should hold good even when savings equal investment.

I am confident that I was right when I made the following comment on this point: "Through the action of credit the exchange value of money for individual goods varies multifariously; hence the relative prices of all consumption goods as well as capital goods must vary accordingly.

Nor can the respective productivities of the latter remain unaltered after the introduction of money into the social

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1) Takata, Theory of Interest, p. 383.

economy, even though we may be prepared to admit that the economic quantities in question are capable of determination without the medium of money, e. g. through exchange in kind. It is, therefore, impossible to maintain that money is merely an economic veil, since, in the case we are considering, the action of money is woven into the structure for the determination of the natural rate of interest, viz., the rate factor which renders equal savings and investment. In other words, the proposition that the rate of interest which would have been formed in real economy would be the money rate of interest when a certain amonnt of capital in the form of savings had been invested cannot be accepted."[2]

It is, of course, questionable whether a unitary rate of interest is possible in real economy, that is, without the interposition of money.[3] A theoretical discussion, however, does not presuppose the actual realization of such a complex system of barter economy, assumption of possibility being sufficient. The question is not whether exchange can actually be carried to the extent suggested with the money factor abstracted, but whether it is feasible to regard it as a limiting element. Let us assume an economy in which exchange is consummated in kind, to as great an extent, let us say, as is possible through the medium of money. In such circumstances there would of course exist definite structures which would function in the determination of the rate of interest.

The Böhm-Wicksellian theory of interest makes this point perfectly clear, though it premises a special and unnatural structure of production. Léon Walras, presupposing the so-called multilineal structure of production[4], makes fairly clear, how the rate of interest is determined where money does not interpose, at a certain stage in the exposition of his

2) ibid., p. 384, Myrdal, Gleichgewichtsbegriff usw., Beiträge zur Geldtheorie, 1933, S. 392.

3) Takata, Theory of Interest, p. 384, Hayek, Geldtheorie u. Konjunkturtheorie, 1929, S. 125.

4) Takata, Theory of Interst, pp. 73—96, pp. 318—320.

equilibrium theory. Still, the question which we have to answer carefully how the interposition of money in such an economic system will affect the rate of interest. The conclusions to be drawn in this regard will vary according to the view that may be taken of the nature of money. Money may be regarded as a mere viel, for instance.

The inevitable conclusion would then be that whatever might happen without the interposition of money would equally occur in money economy, the only difference being that through the interposition relative prices would be converted into corresponding absolute prices, though no amount of monetary interposition would affect the mutual ratios of the absolute prices, viz. relative prices per se. Pointing out that Pareto's system of general equilibrium leaves out money as the circulating medium while taking due note of the measure of value, Divisia has formulated a system of equations to supplement the equation which treats money as a functional medium. In a certain sense his thesis resembles Fisher's equation of exchange. So long as such a line of study is pursued, money will continue to function as a mere economic veil, and as such it can hardly be regarded as affecting the relative values of commodities.

The "cash-balance approach", as it is known, may be said to provide another means of studying the function of money. If this line of investigation is pursued to its logical conclusion, while granting due consideration to the quantity of money which may be hoarded by individual citizens, it will be found that money is by no means a mere veil (something that simply serves to convert relative into absolute value) or that it is nothing more than a multiplying factor. These two methods of approach to a study of the value of money, namely, the income approach (which I have referred

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5) Takata, Study of the Theory of Interest, pp. 142—146, Divisia, Economie rationnelle, 1928, p. 503 et suiv.

6) Marget, Léon Walras and the Cash-Balance Approach, Journal of Political Economy, 1931, p. 569 et seq.

to above and which is sometimes called the quantity theory of money) and this "cash-balance approach," are expounded and utilized by Léon Walras—the former in the first edition of the *Éléments* and the latter in the second edition of the *Éléments* (1889) and later publications. Particularly noteworthy in this respect is his theory of "encaisse désirée"[7], which was carried to a conclusion in the final edition of the *Éléments*. Following the Walras equations let me trace the path by which money really functions, in other words, the way in which the interposition of money affects the value of exchange (relative value) and certain other economic quantities.

2.

I propose to omit any description or explanation of the manner in which the rate of interest is formed where there is no interposition of money, that is to say where exchange in kind or barter alone prevail. I shall here consider, however, the entire system of equations which demonstrates the equilibrium state of money economy, adding such new equations as arise from the consideration of circulation and monetary functions, and as a consequence of capitalization and credit, to the system of equations already constructed to express the equilibrium of exchange and production.

Let A, B, C..... be consumption goods, and T,...P,... K, K', K''..... factors of production, i. e., land, labourers, and fixed capital and the respective services such as services of land, labour and fixed capital goods. Then let M, M'..... be raw materials and goods, let A be the unit of value, p_b, p_c.....the prices of consumption goods, and P_k and $P_{k'}$ the prices of fixed capital goods, and let p_t,...p_p,...p_k...$p_{k'}$ be the

7) Walras, Éléments d'économie politique pure, 1874, p. 180 (according to Marget); Walras, Théorie de la monnaie, 1886; Takuma Yasui, Money and Economic Equilibrium, *Keizaigaku Ronshu* Vol. 8, No. 4.

8) Takuma Yasui, Factor of Time and Capital Interest, *Keizaigaku Ronshu* Vol. 6. Nos. 9 and 10.

prices of the services of the respective factors of production or the prices of production services. Further, let p_m, $p_{m'}$... be the prices of raw materials. It is assumed that money U has no use value in itself and its price p_u is to be expressed by the measure of value A. E denotes the total quantity of the savings of society (the quantity of income in excess of consumption) for a definite period. Furthermore, the storage services of consumption goods a', b'..... must be considered. These are consumption goods which are simply kept in reserve by the producer, viz. the enterprise, and the services as storage of these goods have also their respective prices $p_{a'}$, $p_{b'}$..... Regarding raw materials also, their storage services m'... and price of these services $p_{m'}$ are admissible.

We have thus the total supply equation in regard to each of m number of production services.[9] (Note 1) O_t, O_p, O_k... represent the social supply of each of the production services.

$$\left.\begin{aligned} O_t &= F_t\,(p_t \cdots p_p \cdots p_k\,,\ p_{k'} \cdots p_b\,,\ p_c \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \\ O_p &= F_p\,(p_t \cdots p_p \cdots p_k\,,\ p_{k'} \cdots p_b\,,\ p_c \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \\ O_k &= F_k\,(p_t \cdots p_p \cdots p_k\,,\ p_{k'} \cdots p_b\,,\ p_c \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \\ O_{k'} &= F_{k'}\,(p_t \cdots p_p \cdots p_k\,,\ p_{k'} \cdots p_b\,,\ p_c \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \end{aligned}\right\} \quad (1)$$

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As regards raw materials, the supply equation is omitted, as the quantity held in possession denotes the supply quantity. The same total supply equations may be formed with regard to the storage services of consumption goods as well.[10]

$$\left.\begin{aligned} O_{a'} &= F_{a'}\,(p_t \cdots p_p \cdots p_k\,,\ p_{k'} \cdots p_b\,,\ p_c \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \\ O_{b'} &= F_{b'}\,(p_t \cdots p_p \cdots p_k\,,\ p_{k'} \cdots p_b\,,\ p_c \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \end{aligned}\right\} \quad (1_1)$$

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(Note 1) In Walras' equations of capitalization and credit the total supply quantity of each class of production goods ($O_t \ldots O_p \ldots$) is shown as the function of prices of production services, the prices of consumption goods and p_e.

9) Léon Walras, Éléments, édition définitive, p. 250.
10) ibid., p. 304.

Now, merchandize E denotes ideal goods which may be regarded as "revenu net perpétuel," and the price p_e is, no doubt, $\frac{1}{i}$. In this case, although i represents, in a sense, the rate of interest, it is shown as "taux du revenu net perpétuel"[11]. Such being the case, to be a function of p_e is be a function of i. This is why p_e is replaced here by i. Again, as in the present case, raw materials m, storage services of consumption goods a', b'..., the storage services of raw materials m', money u, and its storage services u' are introduced, the prices of these are regarded as having a bearing on $O_t, \cdots O_p, \cdots$.

Next, as to the total demand function of each class of consumption goods. Concerning the goods B and C..., they are, as already stated, the functions of all prices. With regard to A, the demand for A is represented as the balance left over after the sum of all consumption goods other than A and the savings E have been deducted from the total income; and the total income denotes the sum of the products obtained by multiplying the total quantities of all production services, which the subject as consumer supplies, by their respective prices.

$$\left.\begin{aligned} D_b &= F_b\,(p_t \cdots p_p \cdots p_k \cdots p_b\,,\ p_c \cdots p_m \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \\ D_c &= F_c\,(p_t \cdots p_p \cdots p_k \cdots p_b\,,\ p_c \cdots p_m \cdots p_{a'}\,,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\,,\ i) \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ D_a &= O_t\,p_t + \cdots + O_p\,p_p + \cdots + O_{a'}\,p_{a'} + O_{b'}\,p_{b'} + \cdots + O_u\,p_{u'} \\ &\quad -(D_b\,p_b + D_c\,p_c + \ldots + E) \end{aligned}\right\} \quad (2)$$

Savings E is also the function of the prices of all goods.[12] The quantity obtained by dividing the marginal utility of the portion saved of each individual by the rate of interest i.e. weighted by its price (which means the quantity obtained by multiplying the marginal utility of the ideal revenue goods, as defined above, by the reverse of their prices) being equal to the marginal utility of A, E moves *pari passu* with

11) Walras, op. cit., p. 250.
12) ibid., p. 277.

the prices of other goods.[13]

$$E=F_e\ (p_t \cdots p_b\ ,\ p_c \cdots p_m \cdots p_{a'}\ ,\ p_{b'} \cdots p_{m'} \cdots p_{u'}\ ,\ i) \qquad (3)$$

Let D_k, $D_{k'}$…be the demand for new fixed capital goods and $D_{a'}, D_{b'} \ldots D_m$ ..the demand for newly circulating capital goods. Then, the equality of the supply of and the demand for the services of factors of production is shown as follows :[14] —

$$\left.\begin{array}{l} a_t\ (D_a+D_{a'})+b_t\ (D_b+D_{b'})+\cdots+m_t\ D_m+\cdots \\ \qquad\qquad +k_t\ D_k+k'_t\ D_{k'}+\cdots=O_t, \\ a_p\ (D_a+D_{a'})+b_p(D_b+D_{b'})+\cdots+m_t\ D_m+\cdots \\ \qquad\qquad +k_t\ D_k+k'_t D_{k'}+\cdots=O_p, \\ a_k\ (D_a+D_{a'})+b_k(D_b+D_{b'})+\cdots+m_k D_m+\cdots \\ \qquad\qquad +k_k D_k+k'_k D_{k'}+\cdots=O_k, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{array}\right\} \quad (4)$$

The following equations can similarly be formed with regard to the equality of the supply of and the demand for production services A′, B′…M…[15]

$$\left.\begin{array}{l} a_{a'}\ (D_a+D_{a'})+b_{a'}\ (D_b+D_{b'})+\cdots+m_{a'}\ D_m+\cdots \\ \qquad\qquad +k_{a'}\ D_k+\cdots=O_{a'}, \\ a_{b'}\ (D_a+D_{a'})+b_{b'}\ (D_b+D_{b'})+\cdots+m_{b'}\ D_m+\cdots \\ \qquad\qquad +k_{b'}\ D_k+\cdots=O_{b'}, \\ a_m\ (D_a+D_{a'})+b_m\ (b_b+D_{b'})+\cdots+m_m\ D_m+\cdots \\ \qquad\qquad +k_m\ D_k+\cdots=Q_m, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{array}\right\} \quad (4')$$

The principle that price is equal to the cost of production applies to all consumption goods and raw materials.[16]

$$\left.\begin{array}{l} a_t\ p_t+a_p\ p_p+a_k\ p_k\cdots+a_{a'}\ p_{a'}+a_{b'}\ p_{b'}+\cdots+a_m\ p_{m'}+\cdots \\ \qquad\qquad +a_u p_{u'}=1 \\ b_t\ p_t+b_p\ p_p+b_k\ p_k+\cdots b_{a'}\ p_{a'}+b_{b'}\ p_{b'}+\cdots+b_m\ p_{m'}+\cdots \\ \qquad\qquad +b_u\ p_{u'}=p_b \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ m_t\ p_t+m_p\ p_p+\cdots+m_{a'}\ p_{a'}+m_{b'} p_{b'}+\cdots+m_m\ p_{m'}+\cdots \\ \qquad\qquad +m_u\ p_{u'}=p_m \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{array}\right\} \quad (5)$$

13) ibid., p. 251.
14) ibid., pp. 257 & 305.
15) ibid., p. 305.
16) ibid., p. 307.

The same cost of production principle applies to new fixed capital goods also.[16a)]

$$k_t p_t + k_p p_p + \cdots + k_{a'} p_{a'} + k_{b'} p_{b'} + \cdots + k_m p_{m'} + \cdots + k_u p_{u'} = P_k \quad (6)$$

Some explanation is required as to what a_u, $b_u \ldots m_u \cdots k_u$ in (5) and (6) represent. Now, let $\alpha_{a'}, \alpha_{b'} \ldots \alpha_m \ldots \alpha_k$ be the quantities of A', B'...M...K..., each of which must be prepared in storage taking the form of money (and not taking the form of kind) in order to produce one unit of A goods. That is, let them be the coefficients of production of the storage services of A', B'...in the form of money with regard to the production of A. Similarly, let $\beta_{a'}, \beta_{b'} \ldots \beta_m \ldots \beta_k$ and $\mu_{a'} \mu_{b'}, \mu_m \cdots \mu_k \cdots$ be the coefficients of production of such storage service of money involved in the production of B and M respectively. Then, a_u, for instance, is none other than the sum of the products obtained by multiplying these coefficients of production in the above-mentioned sense, i. e. quantities of the production services of the factors which must be held in the form of money for the production of each product by their respective prices. In short, a_u is the quantity of the storage service of the money required for the production of one unit of A, or, in other words, the cash balance of the firm necessary for the production of one unit of A. Accordingly, we have the following equation :[17)]—

$$a_u = \alpha_{a'} p_{a'} + \alpha_{b'} p_{b'} + \cdots + \alpha_m p_{m'} + \cdots + \alpha_k p_k + \cdots$$
$$b_u = \beta_{a'} p_{a'} + \beta_{b'} p_{b'} + \cdots + \beta_m p_{m'} + \cdots + \beta_k p_k + \cdots$$
$$m_u = \mu_{a'} p_{a'} + \mu_{b'} p_{b'} + \cdots + \mu_m p_{m'} + \cdots + \mu_k p_k + \cdots$$
$$k_u = \chi_{a'} p_{a'} + \chi_{b'} p_{b'} + \cdots + \chi_m p_{m'} + \cdots + \chi_k p_k + \cdots$$

Next, the total amount of savings E is equal in established equilibrium to the total amount of the prices of new capital

16a) ibid., p. 307.
17) ibid., p. 306.

goods (new fixed and circulating capital goods). That is to say, savings are equal to investment.[18]

$$D_k P_k + D_{k'} P_{k'} + \cdots + D_{a'} + D_{b'} p_b + \cdots + D_m p_m + \cdots = E \qquad (7)$$

The equality of the rates of net profit with regard to all fixed capital goods (l'égalité du taux du revenu net) must be in evidence in equilibrium.[19]

$$P_k = \frac{p_k}{i + \mu_k + \nu_k},\ P_{k'} = \frac{p_{k'}}{i + \mu_{k'} + \nu_{k'}}, \cdots\cdots \qquad (8)$$

In this case, μ represents the depreciation rate of fixed capital goods, while ν represents the rate of compensation against risks (premium rate).

The above equation can be derived from the following simple relations. If p reperesents the annual receipts and P the price of fixed capital goods,[20] $p-(\mu+\nu)P=iP$, $P=\frac{p}{i+\mu+\nu}$

Accordingly, so far as K is concerned, we have the following equation: $P_k = \frac{p_k}{i + \mu_k + \nu_k}$

Equality of the rates of profit concerning all kinds of circulating capital goods is shown by the following equation[21].

$$1 = \frac{p_{a'}}{i},\quad p_b = \frac{p_{b'}}{i} \cdots p_m = \frac{p_{m'}}{i} \cdots p_u = \frac{p_{u'}}{i} \qquad (7_1)$$

The equations relative to the demand for and the supply of money should be added to the foregoing. The total supply of money is the balance left over after income cash (encaise désirée, money held by individuals or consumers) has been deducted from the total quantity of money in existence Q_u (i. e. Q_u-encaisse désirée.) This balance ought

18) ibid., pp. 258 & 307.

19) ibid., p. 258; Regarding terms translated, O. Lange, Rate of Interest and the Optimum Propensity to Consume, Economica, Feb, 1938, p. 21, Footnote.

20) ibid., p. 244.

21) ibid., p. 307.

to be equal to the amount of cash required by the enterprise, viz. the business cash. Let the demand quantities of the individuals for the storage services of A′, B′.....in the form of money (not in kind and at a certain fixed price of services) be α, β.....respectively, and the demand quantity in the form of money for the storage services of ideal goods E be ε. Then, the demand of the individuals for such storage services in the form of money, i. e. the size of income cash, is the quotient obtained by dividing the sum of these storage services multiplied by their respective prices by the price of the service of money (the price of money in A multiplied by the rate of interest). Therefore, the money supply in the case of each individual is shown by the following formula :[22]—

As applied to society as a whole, we similarly have the following :—

$$O_u = Q_u - \frac{d\alpha p_{a'} + d\beta p_{b'} + \cdots + d\varepsilon p_{a'}}{p_{u'}} \qquad (9)$$

Next, the storage services of A in money form, that is, A′ in the form of money is in demand. Let it be δ_a. As already noted, $\alpha_{a'}$ is the quantity of the service of money, as storage which is required for the production of one unit of A. The total demand for A is equal to the aggregate of the demand for it as consumption goods D_a, and the demand for it as new circulating capital goods $D_{a'}$. Accordingly, $\alpha_{a'}(D_a + D_{a'})$ is the service of the total monetary storage representing the purchasing power for A′ which must be held in such form for the production of A. Similarly, $B_{a'}(D_b + D_{b'})$ is the monetary service as storage with regard to A′ which is necessary for the production of B. The aggregate of these (i. e. of $\alpha_{a'}(D_a + D_{a'}) + \beta_{a'}(D_b + D_{b'}) + \cdots$) is δ_a, to which reference has already been made. Similarly as to $\delta_\beta \cdots \delta_\mu \cdots$[23]

$$\alpha_{a'}(D_a + D_{a'}) + \beta_{a'}(D_b + D_{b'}) + \cdots + \mu_{a'} D_m + \cdots + \chi_{a'} D_k + \cdots = \delta_\alpha$$
$$\alpha_{b'}(D_a + D_{a'}) + \beta_{b'}(D_b + D_{b'}) + \cdots + \mu_{b'} D_m + \cdots + \chi_{b'} D_k + \cdots = \delta_\beta$$
.....

22) ibid., 305.

23) ibid., p, 309.

$$\alpha_m (D_a + D_{a'}) + \beta_m (D_b + D_{b'}) + \cdots + \mu_m D_m + \cdots + \chi_m D_k + \cdots = \delta_\mu$$

$$\alpha_k (D_a + D_{a'}) + \beta_k (D_b + D_{b'}) + \cdots + \mu_k D_m + \cdots + \chi_k D_k + \cdots = \delta_\chi$$

Starting on this premise, the total cash of firms or the total quantity of the service of money as storage required for production will be shown by the numerator on the left side of the following formula. This represents the sum of the storage services of A, B.....which are required in the form of money multiplied by the prices of their services. By dividing this sum by the price of the service of the unit of money we find the quantity of the money in storage which is necessary for the production, or the demand for money on the part of the firms concerned.[24]

$$\frac{\delta_\alpha p_{a'} + \delta_\beta p_{b'} + \cdots + \delta_\mu p_{m'} + \cdots + \delta_\chi p_k + \cdots\cdots}{p_{u'}} = O_u \qquad (10)$$

What is deducted from Q_u on the right side of (9) is the cash-balance of individuals, while the left side of (10) shows the demand for money on the part of the enterprises i. e. the money held by firms. The aggregate of these two must be Q_u in equilibrium. If (9) and (10) are replaced by the equation showing the equality of the aggregate of cash of the two kinds to Q_u, the demand quantity of money O_u can be deleted from among the numbers of the unknown quantities.

So far, I have linked the equations of capitalization and credit with those of circulation and money (Walras' chapters 24 and 29) in preparation to the derivation of the function of money from them. I shall now make a general survey of this system of equations.

As a necessary preliminary, let me first examine the relation between the equations and the number of the unknown quantities in the capitalization and credit equations.[25]

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24) ibid., p. 307.

25) ibid., p. 256.

(1) Supply of production services:

$$O_t = F_t(p_t \cdots p_p \cdots p_k, p_{k'} \cdots p_b, p_c \cdots p_e)$$
$$O_p = F_p(p_t \cdots p_p \cdots p_k, p_{k'} \cdots p_b, p_c \cdots p_e)$$
$$O_k = F_k(p_t \cdots p_p \cdots p_k, p_{k'} \cdots p_b, p_c \cdots p_e)$$

(2) Demand for consumption goods:

$$D_b = F_b(p_t \cdots p_p \cdots p_k, p_{k'}, p_b, p_c \cdots p_e)$$
$$D_c = F_c(p_t \cdots p_p \cdots p_k, p_{k'}, p_b, p_c \cdots p_e)$$
$$D_a = O_t p_t + \cdots + O_p p_p + \cdots + O_k p_k + O_{k'} p_{k'} + \cdots - (D_b p_b + D_c p_c + \cdots + E)$$

(3) Function of savings:

$$E = F_e(p_t \cdots p_p \cdots p_k, p_{k'} \cdots p_b, p_c \cdots i)$$

(4) Equality of supply of and demand for production services:

$$O_t = a_t D_a + b_t D_b + \cdots + k_t D_t + k'_t D_{k'} + \cdots$$
$$O_p = a_p D_a + b_p D_b + \cdots + k_p D_k + k'_p D_{k'} + \cdots$$
$$O_k = a_k D_a + b_k D_b + \cdots + k_k D_k + k'_k D_{k'} + \cdots$$

(5) Cost of production principle for consumption goods:

$$a_t p_t + \cdots + a_p p_p + \cdots + a_k p_k + \cdots = 1$$
$$b_t p_t + \cdots + b_p p_p + \cdots + b_k p_k + \cdots = p_b$$

(6) Cost of production principle for fixed capital goods:

$$k_t p_t + \cdots + k_p p_p + \cdots + k_k p_k + \cdots = P_k$$
$$k'_t p_t + \cdots + k'_p p_p + \cdots + k'_k p_{k'} + \cdots = P_{k'}$$

(7) Equality of the prices of new capital goods to savings.

$$D_k P_k + D_{k'} P_{k'} + \cdots\cdots = E$$

(8) Equality of the rates of net profit with regard to various capital goods:

$$P_k = \frac{p_k}{i + \mu_k + \nu_k}, \quad P_{k'} = \frac{p_{k'}}{i + \mu_{k'} + \nu_{k'}}, \cdots\cdots$$

The number of unknown quantities is n in the supply quantitites of services, n in the prices of services, m in the demand quantities of products (consumption goods), m-1 in

the quantity of new fixed capital goods, one in their price, one in the quantity of savings and one in the rate of interest making a total of $2n+2m+2l+1$. The number of equations is n in (1), m in (2), one in (3), n in (4), m in (5), one in (6) and one in (8), making a total of $2n+2m+2l+2$. However, one in (2), (4), (5) and (6) can be derived from other equatitions, so that the number of unknown quantities and the number of equations are equal.

Now, let us compare the unknown quantities and equations newly introduced in the development of the system of circulation and money equations.

The number of equations newly added is m in (1′), m and s (the kind of raw materials m, m′.....is shown by s) in (4′), m plus s and one (concerning p_u) in (8′) and one each in (9) and in (10), making a total of $3m+2s+3$. On the other hand, the number of unknown quantities newly added is $m+1$ in the quantities, exchanged the services of circulating capital goods (A′, B′……) and of the services of money U, $m+s+1$, in the prices of the services of circulating capital goods, of the services of money and of raw materials and $m+s+1$ in the production quantities of capital goods and raw materials, making a total of $3m+2s+3$. The number of unknown quantities and the number of equation newly added are thus equal, and so the value of these unknown quantities can be unequivocally determined.[26]

Let us further consider the equations (9) and (10) regarding the supply of and the demand for money. By putting in juxtaposition the right side of (9) and the left side of (10), we have the following formula :—

$$Q_u - \frac{d_\alpha p_{a'} + d_\beta p_{b'} + \cdots + d_\varepsilon p_{n'}}{p_{u'}} = \frac{\delta_\alpha p_{a'} + \delta_\beta p_{b'} + \cdots + \delta_\mu p_{m'} + \cdots + \delta_\chi p_k}{p_{u'}}$$

Now, let D_α, A_α and E_α be such as are given below. Then, the sum of these three H_α, divided by the price of the

26) ibid., p. 308.

service of money as storage $P_{u'}$ must be equal to the quantity of money Q_u.

$$d_\alpha p_{a'} + d_\beta p_{b'} + \cdots = D_\alpha,$$
$$\delta_\alpha p_{a'} + \delta_\beta p_{b'} + \cdots + \delta_\mu p_{m'} + \delta_\chi p_k + \cdots = \Delta_\alpha,$$
$$d_\varepsilon p_{a'} = E_\alpha,$$
$$D_\alpha + \Delta_\alpha + E_\alpha = H_\alpha;$$
$$Q_u = \frac{H_\alpha}{p_{u'}}$$

By dividing D_α, Δ_α and E_α each by $P_{u'}$, we find the currencies held by the consumers, those held by the producers and those held as savings respectively.[27]

3.

I now propose to derive from such an exposition of Walras' theories an insight into how the action of money ought to be regarded. To this end, however, I will first try to simplify Walras' too complex system of equations by means of certain abstractions. Let me abstract raw materials, to begin with, and then the storage services of consumption goods. This we can do by directing attention to circulating equilibrium instead of to what is called constructive equilibrium.[28] The money held by the producers (cash available to firms) may well be regarded as the money provided against the necessity of purchasing production services. The postulate that the unit of money does not operate as the measure of value will be retained as it is.

(1) Supply of production services:

$$O_t = F_t(p_t, p_p \cdots p_b, p_c \cdots p_{u'}, i)$$
$$O_p = F_p(p_t, p_p \cdots p_b, p_c \cdots p_{u'}, i)$$
$$O_k = F_k(p_t, p_p \cdots p_b, p_c \cdots p_{u'}, i)$$

(2) Demand for consumption goods:

27) ibid., p. 310; A. W. Marget, op. dit., p. 582.
28) Takuma Yasui, op. cit., p. 52.

$$D_b = F_b(p_t, p_p \cdots p_b, p_c \cdots p_{u'}, i)$$

$$D_a = O_t p_t + O_p p_p + O_k p_k + O_{k'} p_{k'} + \cdots + O_u p_{u'} - (D_b p_b + D_c p_c + \cdots + E)$$

(3) Function of savings:

$$E = F_e(p_t, p_p \cdots p_b, p_c \cdots p_{u'}, i)$$

(4) Equality of supply of and demand for production services:

$$a_t D_a + b_t D_b + \cdots + k_t D_k + \cdots = O_t$$
$$a_p D_a + b_p D_b + \cdots + k_p D_k + \cdots = O_p$$
$$a_k D_a + b_k D_b + \cdots + k_k D_k + \cdots = O_k$$

(5) Cost of production principle concerning consumption goods:

$$a_t p_t + a_p p_p + \cdots\cdots + a_u p_{u'} = 1$$
$$b_t p_t + b_p p_p + \cdots\cdots + b_u p_{u'} = p_b$$

(6) Cost of principle concerning fixed capital goods production:

$$k_t p_t + k_p p_p + \cdots\cdots + k_u p_{u'} = P_k$$
$$k'_t p_t + k'_p p_p + \cdots\cdots + k'_u p_{u'} = P_{k'}$$

(7) Equality of investment and savings:

$$D_k P_k + D_{k'} P_{k'} \cdots\cdots = E$$

(8) Equality of the rates of net profit concerning various capital goods:

$$P_k = \frac{p_k}{i + \mu_k + \nu_k},\quad P_{k'} = \frac{p_{k'}}{i + \mu_{k'} + \nu_{k'}}, \cdots p_u = p_u = \frac{p_{u'}}{i}$$

(9) Supply of money:

$$O_u = Q_u - \frac{d\alpha p_{a'} + d\beta p_{b'} + \cdots\cdots d\varepsilon p_{a'}}{p_{u'}}$$

(10) Demand for money(note):

$$O_u = \frac{\delta t\, p_{t'} + \delta p\, p_{p'} + \cdots + \delta k\, p_k + \cdots}{p_{u'}}$$

(Note) Walras' equation corresponding to this formula contains the storage services of consumption goods in the

form of money as δ_α, δ_β..... But enterprise provides money for the purchase of production services and accordingly if the coefficients of production services in the form of money are shown a_ι, a_ρ, a_κ.....may supplant δ_α, δ_β..... in equation (9). Then, we have (10) as the equation showing business cash. Mr. Takuma Yasui's revision in this regard must be accepted in its entirety[29] Walras' point of view both in regard to the nature and action of raw materials and concerning the relation between money in saving and the total quantity of saving calls for some critical study, but I am unable to devote leisure to the elucidation of these points in detail at this juncture.

The number of equations is n in (1), m in (2), one in (3), $n-1$ in (4), m in (5), one in (6), one in (7), $1-1$ in (8), and one each in (9) and in (10), but as one, whichever we may choose, contained in (2), (5), (5) and (6) can be derived from other expressions, it can be treated as not independent. Thus, the total number of equations is $2m+2n+21+5$. On the other hand, the number of unknown quantities is n each in the exchange quantities of production services and in their prices, m each in the exchange quantities of consumption goods and in their prices, one each in the quantities of new fixed capital goods and in their price, one each in the price of money and in the price of its service as storage, one in the supply and demand quantity of money, one in the rate of interest, and one in the quantity of savings, making a total of $2m+2n+21+5$.

In this case, does money serve simply as an economic veil? Does it, in other words, operate as a mere coexistent phenomenon incapable of influencing exchange value in any way? In my opinion, it is impossible to think so. Confining attention to the last expression of equations (2), it will be seen that it contains, where money interposes, the money rate of interest $O_u p_{u'}$ as surplus purchasing power or income. There is no ground for believing that the demand for A is

29) ibid., p. 71.

the same whether money interposes into the system or not. Nay, D_b, D_c.....being here the function of p_a or $p_{a'}$, ought to acquire numerical value different from what they would have were there no interposition of money. This point may be elucidated more clearly if we consider the matter while taking into consideration Pareto's system of equations which starts from the economic quantities of goods at the beginning of the term and accordingly of the quantity of money in possession before equilibrium. The equations in (5) and (6) contain also the rate of interest of money held by firms, so that we can conceive that exactly the same relative prices will be formed as in cases where it is not contained as an element of cost. Moreover, since there is the interposition of such elements as the interest of money held in storage by firms, the relation between the prices of products and the prices of productive services must necessarily be different from that which would exist between them if money did not enter into the system of economy.

It is only fair to assume that the interposition of the factor of money not only materially affects the relative prices but influences the rate of interest as well. Naturally it is going too far to say as is implied in Keynes' theory of interest, that the rate of interest is determined by monetary considerations only. Inasmuch as Walras deals with a sort of static state which involves at the same time an implication of a transition to the next enlarged static state,[30] while Keynes deals with a short period equilibrium, their theories do not coincide so far as the above mentioned problem is concerned. Nevertheless, the views which they hold regarding the action of money in determining the rate of interest are not so divergent as to forbid all comparison.

Keynes stresses the action of interference which money can exert on the rate of interest. Now take (9) and (10) out of the group of equations already given. If all other conditions are the same, the rate of interest and the demand

30) Walras. op. cit., pp. 298 & 302.

and supply of money must be determined by the relation of (9) and (10). If (10) is deducted from (9), the next equation $Q_{n}=\frac{H_{\alpha}}{p_{n'}}$ can be arrived at. The right side of this equation represents, as already noted, the real balance held by the enterprises and consumers, and it tends to be larger, in proportion as the rate of interest lost through the hoarding of money happens to be lower. If, therefore, the other variables are unchanged, the rate of interest partakes of the nature of the decreasing function of Q_{n}. But this is the conclusion drawn from the equations concerning money in (9) and (10) only, on the hypothesis that all conditions other than the rate of interest, the quantity of money and the cash balance are fixed. A different conclusion will be reached if other factors which also operate to determine the rate of interest are taken into due account and various other equations are accordingly considered at the same time. Similarly, if the equations (3) and (7) only are singled out and all other conditions are abstracted (that is, if they are regarded as fixed), we have the following result. The left side of (7) is the decreasing function of the rate of interest, while the right side of (3) is its increasing function[31] (decreasing, after a certain point is reached). The former represents investment and the latter savings. The rate of interest is determined where these two are equal. This is exactly what the classical theory of interest contends, but this theory holds true only where it is premised that all other conditions are the same. Consequently, when (9) and (10) are taken into consideration as well, the conclusion is obvious that the rate of interest is determined by monetary influences.

From this point of view, both the theory that the rate of interest is determined by monetary conditions only and the one which contends that it is determined by the demand and supply of savings cover only one special aspect of the

31) Takuma Yasui, op. cit., p. 82.

so-called general theory of interest as already indicated by Prof. Lange and others; this holds true even when we do not take into consideration the creation of credit, which implies evidently the material interference of money on the economic system. Thus the thesis which refuses to attach any more significance to money than to regard it as a multiplicative factor in the formation of absolute prices cannot be accepted as sufficiently convincing.